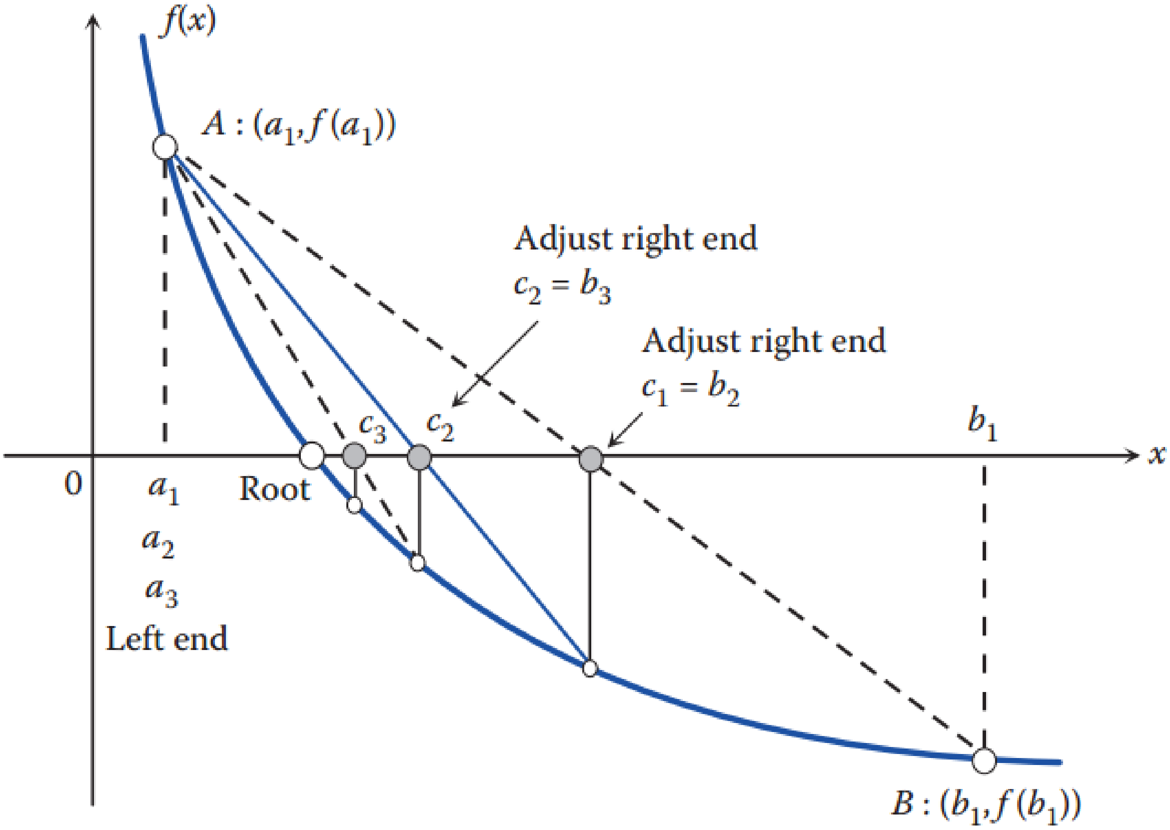
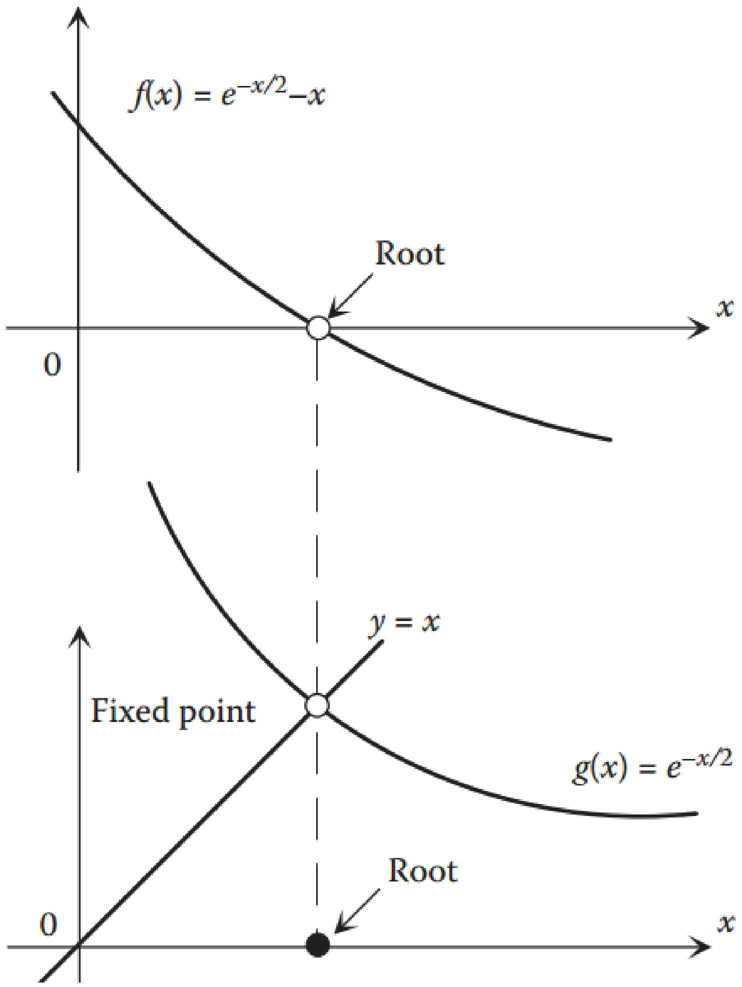
REGULA FALSI METHOD



3.4 Fixed-Point Method

The fied-point method is an open method to find a root of $f(x) = 0$. The idea is to rewrite $f(x) = 0$ as $x = g(x)$, where $g(x)$ is called the iteration function.



$$x_{n+1} = g(x_n), \quad n = 1, 2, 3, \ldots, \quad x_1 = \text{initial guess}$$

$$|x_{n+1} - x_n| < \varepsilon$$

if $|g'(x) < 1|$ near a fixed point of $g(x)$, convergence is guaranteed.

$$x^2 - 4x + 1 = 0$$

$$x^2 - 4x + 1 = 0 \quad \Rightarrow \quad x^2 = 4x - 1 \quad \Rightarrow \quad x = \frac{4x-1}{x} = g(x)$$

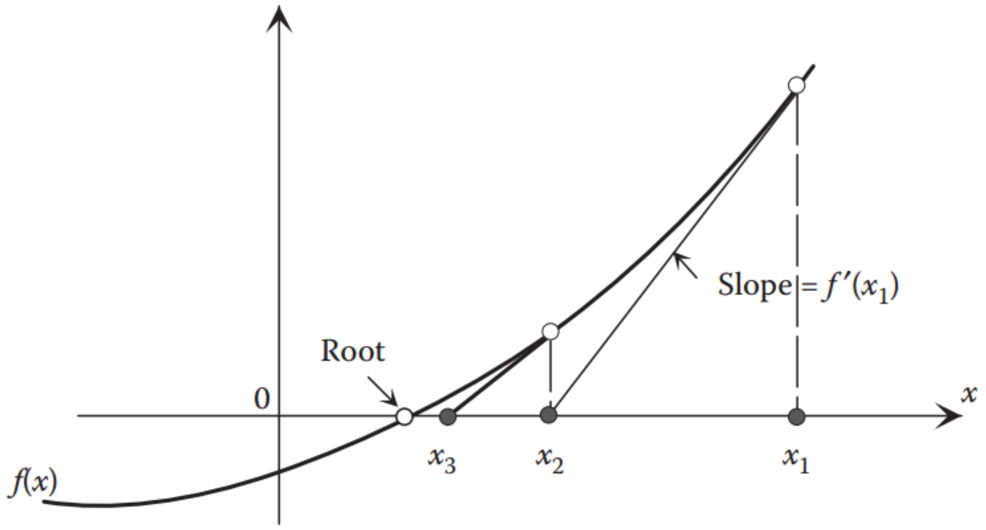
$$\Rightarrow \quad \frac{4x-1}{x} = g(x) \quad \Rightarrow \quad g'(x) = \frac{1}{x^2} < 1 \quad \Rightarrow \quad |x| > 1$$

$$x^2 - 4x + 1 = 0 \quad \Rightarrow \quad x^2 + 1 = 4x \quad \Rightarrow \quad x = \frac{x^2+1}{4} = g(x)$$

$$\Rightarrow \quad \frac{x^2+1}{4} = g(x) \quad \Rightarrow \quad g'(x) = \frac{x}{2} < 1 \quad \Rightarrow \quad |x| < 2$$

$$g_1(x) = \frac{1}{4}(x^2+1), \quad g_2(x) = 4 - \frac{1}{x}$$

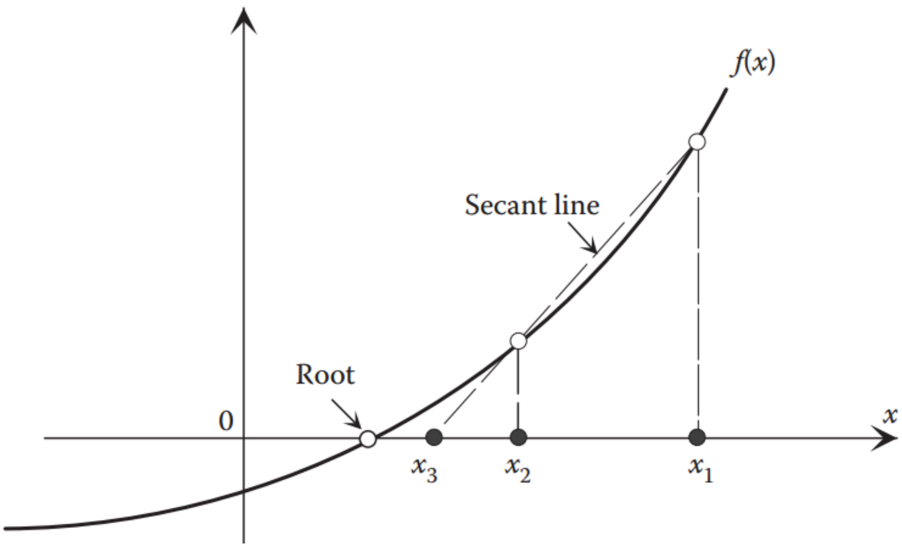
Newton's Method (Newton −Raphson Method)



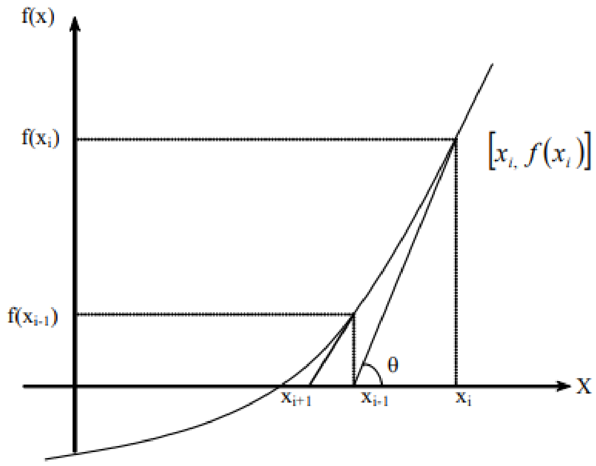
$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \ldots, \quad x_1 = \text{initial point}$$

Secant Method

$$x_{n+1} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n), \quad n = 2, 3, 4, \ldots, \quad x_1, x_2 = \text{initial points}$$



Secant Method



Newton's Method

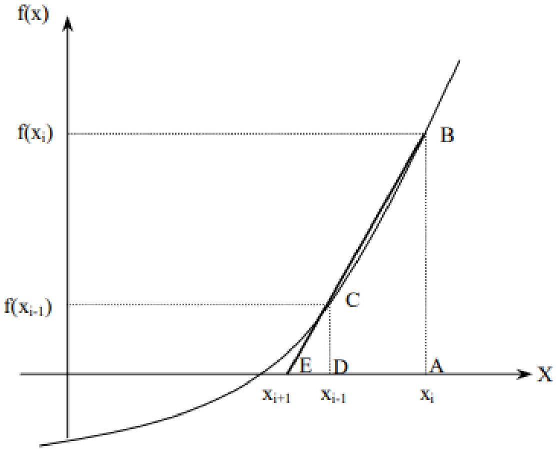
$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

Approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Secant Method



Similar Triangles

$$\frac{AB}{AE} = \frac{DC}{DE}$$

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$