HW # 1

Obtain the root of each equation in the indicated interval by using

Bisection Method

1) $\cos 2x + \frac{2}{3}\sin x = 0,\ [0,2]$

```
clear
a = 0; b = 2; tol = 10^(-6);
% y = f(x) = cos(2*x)+(2/3)*sin(x) = 0
while abs(b-a)>tol
fa = cos(2*a)+(2/3)*sin(a);   fb = cos(2*b)+(2/3)*sin(b);
   c = (a+b)/2;   fc = cos(2*c)+(2/3)*sin(c);
     if fa*fc<0
         b=c;
     else
         a=c;
     end
end
[a b ] = [1.104300498962402  1.104301452636719]
abs(b-a) = 0.000000953674316

===============   o   ===============
```

2) $1 + \cos x \cosh x = 0,\ [-5, -4]$

```
x=-5:0.0001:-4;  y =1+cos(x)*cosh(x); plot(x,y),grid
```

```
clear
a = -5; b = -4; tol = 10^(-6);
% y = f(x) = 1 + cos(x)*cosh(x) = 0
while abs(b-a)>tol
fa =1 + cos(a)*cosh(a);    fb =1 + cos(b)*cosh(b);
   c = (a+b)/2;    fc =1 + cos(c)*cosh(c);
     if fa*fc<0
         b=c;
     else
         a=c;
     end
end
[a b ] = [-4.694091796875000  -4.694090843200684]
abs(b-a) = 9.536743164062500e-07
```

=============== o ===============

3) $\frac{1}{3}x + \ln x = 1,\ [1,2]$

```
x=1:0.0001:2;  y = x*(1/3) + log(x) -1; plot(x,y),grid
```

```
clear
a = 1; b = 2; tol = 10^(-6);
% y = f(x) = x*(1/3) + log(x) -1 = 0
while abs(b-a)>tol
fa = a*(1/3) + log(a) -1;    fb = b*(1/3) + log(b) -1;
   c = (a+b)/2;    fc = c*(1/3) + log(c) -1;
     if fa*fc<0
         b=c;
     else
         a=c;
     end
end
[a b] = [1.596519470214844   1.596520423889160]
abs(b-a) = 9.536743164062500e-07
```

=============== o ===============

Regula Falsi Method

$$e^{-2x/3} + \ln\left(\tfrac{1}{2}x\right) = 0,\ [1, 2]$$

$$x^3 - 5x + 3 = 0,\ [1, 2]$$

$$x^2 + e^{x/2} = 5,\ [1, 2]$$

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Fixed-Point Method

$$x_{n+1} = g(x_n), \quad n = 1, 2, 3, \dots, \quad x_1 = \text{initial guess}$$

$$3x^2 + 2.72x - 1.24 = 0$$

$$g_1(x) = \frac{-3x^2 + 1.24}{2.72}, \quad g_2(x) = \frac{-2.72x + 1.24}{3x}$$

=============== o ===============

Newton's Method

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 1, 2, 3, \dots, \quad x_1 = \text{initial point}$$

$$\ln\left(\tfrac{1}{3}x + 1\right) = 2x + 1,\ [-1, 0]$$

$$f(x) = \ln(1 + x/3) - 2x - 1 = 0 \quad \Rightarrow \quad f'(x) = \frac{1/3}{1 + x/3} - 2 \quad \Rightarrow \quad f'(x) = \frac{1}{3 + x} - 2$$

$$\Rightarrow \quad f'(x) = -\frac{2x + 5}{3 + x}$$

$$x(n+1) = x(n) - \frac{f(x)}{f'(x)} \quad \Rightarrow \quad x(n+1) = x(n) - \frac{[\ln(1 + x/3) - 2x - 1]}{-\frac{2x + 5}{3 + x}}$$

```
x = -1:0.001:0; y = log(1+x/3)-2*x-1; plot(x,y),grid
```

```
clear
x(1)=-0.5;
for k=1:10
    y = log(1+ x(k)/3)-2* x(k)-1;
    y_der = -(2*x(k)+5)/(x(k)+3);
    x(k+1)= x(k)-(y/y_der),pause
end
```

ans : x = -0.614628760162253

=============== o ===============

$$e^{-(x-1)} = 2.6 + \cos(x + 1), \ [-1, 1]$$

$$f(x)=e^{-x+1}-\cos(x+1)-2.6=0 \quad \Rightarrow \quad f'(x)=-e^{-x+1}+\sin(x+1)$$

$$x(n+1)=x(n)-\frac{f(x)}{f'(x)} \quad \Rightarrow \quad x(n+1)=x(n)-\frac{\left[e^{-x(n)+1}-\cos(x(n)+1)-2.6\right]}{-e^{-x(n)+1}+\sin(x(n)+1)}$$

```
x = -1:0.001:1; y = exp(-x+1)-2.6-cos(x+1); plot(x,y),grid
clear
x(1)=-0.5;
for k=1:10
    y = exp(-x(k)+1)-2.6-cos(x(k)+1);
    y_der = - exp(-x(k)+1)+sin(x(k)+1);
    x(k+1)= x(k)-(y/y_der),pause
end
```

ans: x = -0.190942509905312

=============== o ===============

$\sin x \sinh x + 1 = 0, [3, 4]$

```
x=-6.5:0.001:6.5;y=sin(x).*sinh(x)+1;plot(x,y),grid
```

```
clear
x(1)=3;
for k=1:30
    y=sin(x(k))*sinh(x(k))+1;
    y_der = cos(x(k))*sinh(x(k))+ cosh(x(k))*sin(x(k));
    x(k+1)= x(k)-(y/y_der),pause
end
```